**Two-Phase Simulation On Natural Convection Of A Nanofluid Along An Isothermal Vertical Plate**

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**Abstract**

*A numerical algorithm is presented for the laminar natural convection flow of a nanofluid along with an isothermal vertical plate. Nanofluid is treated as a two-component mixture as per Boungiorno model, and as such, the effects of Brownian motion and thermophoresis is incorporated. The equations governing the flow are higher-order nonlinear partial differential equations, and subsequently, they are transformed into a set of nonlinear ordinary differential equations using similarity transformation. Finally, they are reduced to a first-order system and we integrate them using Newton Raphson and adaptive Runge-Kutta methods. For the whole numerical procedure, computer codes are developed in the Matlab environment. We compute dimensionless stream function (s), longitudinal velocity (s′), temperature (θ), and nanoparticle volume fraction (f) and illustrate them graphically for various values of five pertinent dimensionless parameters, namely, Prandtl number (Pr), Lewis number (Le), buoyancy-ratio parameter (Nr), Brownian motion Parameter (Nb), and thermophoresis parameter (Nt). The reduced Nusselt number (Nur) is found to be a decreasing function of each of Nr (buoyancy-ratio parameter), Nb (Brownian motion parameter), and Nt (Thermophoresis parameter). The results of the present simulation agree*

Keywords : Brownian Motion, Isothermal Vertical Plate, Nano Fluid, Natural Convection, Thermophoresis, Two-Phase Model.

**Nomenclature**

a1, a2, a3 initial values eq (22)

DBBrownian diffusion coefficient  *D*Tthermophoretic diffusion coefficient.

f1,f2, f3 functions defined in eqs (23, 24)

*g* is the acceleration due to gravity, m/s2

k conductivity of the nanofluid, w/m.k

Le Lewis number, dimensionless

Nb Brownian motion parameter, dimensionless

Nr buoyancy-ratio parameter, dimensionless

Nt thermophoresis parameter, dimensionless

Nur reduced local Nusselt number, dimensionless

Nux Local Nusselt number, dimensionless

Pr Prandtl number, dimensionless

T temperature, K

T∞  free streams temperature, K

u velocity component in x, m/s

v velocity component in y, m/s

x coordinate from the leading edge, m

y coordinate normal to plate, m

z1, z2, z3, z4, z5 variables, eq (19)

**Greek Symbols**

θ function defined in eq (11), dimensionless

α thermal diffusivity, m2/s

μ dynamic viscosity, N.s/m2

η similarity variables, defined in eq (11)

ψ stream function, m2/s

ρ density, kg/m3

φ nanoparticle volume fraction  
φ∞ nanoparticle volume fraction far away from the plate   
ρf is the density of the base fluid, kg/m3   
ρpis the density of the nanoparticles, kg/m3  
β volumetric expansion coefficient of the nanofluid, 1/K  
(ρc)f the heat capacity of the fluid  
(ρc)p the effective heat capacity of the nanoparticle material

**I. Introduction**

Nanofluids are a relatively new class of fluids which consists of base fluid with nano-sized particles suspended. Naphon and Nakharintr [VIII] proposed three mathematical models for nanofluid: single-phase approach, mixture two-phase approach, and volume of fluid (VOF) approach. Two phase model was proposed by Boungiorno [VII], who analyzed seven mechanisms between nanoparticles and base fluid. Brownian diffusion and thermophoresis predominate over the other five mechanisms, namely, inertia, diffusiophoresis, Magnus effect, fluid drainage, and gravity. The classical problem of the natural convective boundary-layer flow of a nanofluid along an isothermal vertical plate was studied by Kuznetsov and Nield [IV]. Khan and Aziz [IX] worked on it for the uniform heat flux boundary condition. The present paper aims to develop a simple efficient numerical algorithm for laminar natural convection flow of a nanofluid along with an isothermal vertical plate.

**II. Mathematical Model**

The natural convection flow is steady, laminar, two-dimensional. The X-axis is along with the vertical plate, and the y-axis normal to the plate. The temperature T and nanoparticle fraction φ at the plate are constant values, Tw and φw respectively. The ambient values are T∞ and φ∞ respectively. Following Oberbeck-Boussinesq approximation, the equations governing the flow [VII, V, VI] are:

 (1)

 (2)

 (3)

 (4)

 (5)

With the following boundary conditions:

At y = 0: u = v = 0, T = Tw, φ = φw

For large y: u = v = 0, T = T∞, φ = φ∞ (6)

We introduce the stream function (ψ):

 (7)

The continuity equation (1) is automatically satisfied with (ψ). We can eliminate p from Eqs (2) and (3) by cross-differentiation, and get the following equations

 (8)

 (9)

 (10)

Where is the thermal diffusivity of the fluid.

Armed with the following similarity variable and dimensionless variables







 (11)

equations (8), (9) and (10) may be rewritten as (with a prime denoting differentiation with respect to η)

 (12)

 (13)

 (14)

The appropriate boundary conditions are:



 (15)

The buoyancy-ratio parameter (Nr), Brownian motion parameter (Nb), thermophoresis parameter (Nt), local Rayleigh number Rax, Prandtl number (Pr), and Lewis number (Le) are defined as follows:











 (16)

Local Nusselt number is

 (17)

where qw is the wall heat flux. The reduced local Nusselt number is then

 (18)

**III. Solution Procedure**

Eqs (8) (9) and (10) are coupled nonlinear ordinary differential equations. Out of seven, there are three unknown initial values: , and .

***Reduction to First-Order System***

We do this by defining new variables:













 (19)

Eqs (12), (13), and (14) can then be written as













 (20)

The boundary conditions are then:

 (21)

***Solution to Initial Value Problems***

To solve eqs (20), we denote the three unknown initial values by a1, a2, and a3, the set of initial conditions is then:

 (22)

If we solve eqs (20) with adaptive Runge-Kutta method using the initial conditions eq (21), then computed boundary values at  depending on  respectively:

 (23)

The proper values of a1, a2 and a3 offer boundary conditions at ; that is, they suit the equations





 (24)

These nonlinear algebraic equations are solved by the Newton-Raphson method. A value of 15 works well for infinity, even if we integrate further no significant change will occur.

**Code Details**

We utilize Newton Raphson and adaptive Runge-Kutta methods and develop a set of Matlab routines for the solution of eqs (20) along with the boundary conditions (22). These are shown in Table 1.

**Table 1: Matlab routines run sequentially to solve Equations (20)**

|  |  |
| --- | --- |
| **Matlab code** | **Brief Description** |
| deqs.m | Defines the differential equations (20) |
| incond.m | Describes initial values for integration, a1, a2, and a3 are guessed values, eq (22) |
| runKut5.m | Integrates as initial value problem using adaptive Runge-Kutta method |
| residual.m | Yields boundary residuals and approximate solutions |
| newtonraphson.m | Provides correct values a1, a2, and a3 using approximate solutions from residual.m |
| runKut5.m | Again integrates eqs (20) using correct values of a1, a2, and a3. |

The above-mentioned codes confer the tabulated values of as functions of η.

**IV. Results and Discussion**

**Agreement with published works.**

As validation, we compare numerical results obtained from the abovementioned codes with published papers [VIII, VI] in Table 2 and perceive in very good agreement.

**Table 2: Comparison of the present reduced Nusselt number (Nur) values when Le = 10, Nr = Nb = Nt = 10-5**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Pr** | 1 | 10 | 100 | 1000 |
| **Nur [7]** | 0.401 | 0.465 | 0.490 | 0.499 |
| **Nur [3]** | 0.401 | 0.463 | 0.481 | 0.484 |
| **Nur [present]** | 0.401 | 0.465 | 0.486 | 0.490 |

**An Illustrative Case**

We run the codes for Pr = 10, Le = 10, Nr = Nb = Nt = 0.5 and present the results in terms of distributions of dimensionless stream function (s), longitudinal velocity (s′), temperature (θ) and nanoparticle volume fraction (f) in Fig. 1, and it agrees well with Fig. 1 in A. V. Kuznetsov & D. A. Nield [IV].



**Fig. 1:** Plots of dimensionless similarity functions s(η), s′(η), θ (η), f(η) for the case Pr = 10, Le = 10, Nr = Nb = Nt = 0.5.

**Effects of parameters Le, Pr, Nr, Nt, and Nb**

Computations have been performed with the codes to explore the effects of five dimensionless parameters, namely, Prandtl number (Pr), Lewis number (Le), buoyancy-ratio parameter (Nr), Brownian motion Parameter (Nb), and thermophoresis parameter (Nt) on the solution.

Figures 2 and 3 show the influence of Lewis number (Le) on the stream function (s), longitudinal velocity (s′), temperature (θ) and nanoparticle volume fraction (f) for Nr = Nt = Nb = 0.5 and Pr = 1 and Nr = Nt = Nb = 0.5 and Pr = 100 respectively.



(a) on-stream function, s



(b) on velocity function, s′



(c) on temperature function, θ



(d) on nanoparticle volume fraction, f

Fig 2. Effects of Lewis number (Le) on various functions with Nr = Nt = Nb = 0.5 and Pr = 1



(a) on-stream function, s



(b) on velocity function, s′



(c) on temperature function, θ



(d) on nanoparticle volume fraction, f

**Fig. 3:** Effects of Lewis number (Le) on various functions with Nr = Nt = Nb = 0.5 and Pr = 100

Figures 4 depicts the effect of Prandtl number (Pr) on the stream function (s), longitudinal velocity (s′), temperature (θ) and nanoparticle volume fraction (f) for Nr = Nt = Nb = 0.5 and Le = 10.



(a) on stream function, s



(b) on velocity function, s′



(c) on temperature function, θ



(d) on nanoparticle volume fraction, f

**Fig. 4:** Effects of Prandtl number (Pr) on various functions with Nr = Nt = Nb = 0.5 and Le = 10

The dual weight of the buoyancy-ratio parameter (Nr), Brownian motion Parameter (Nb), and thermophoresis parameter (Nt) on the solution for Pr = Le = 10 are shown in Fig. 5.



on stream function, s



(b) on velocity function, s′

****

(c) on temperature function, θ

****

(d) on nanoparticle volume fraction, f

**Fig. 5:** Joint effects of buoyancy ratio (Nr), Brownian motion parameter (Nb) and thermophoresis parameter (Nt) on various functions with Pr = 10 and Le = 10.

**Correlation**

The codes developed in the present study, have been run for values of Nr, Nb, Nt in the range [0.1, 0.2, 0.3, 0.4, 0.5] with Pr = 10 and Le = 10, and the reduced Nusselt number (eq. 18) is calculated. We perform linear regression on the results and get the following correlation

Nurest = 0.467 – 0.0057Nr – 0.254Nb – 0.161Nt (25)

As per eq (25), the reduced Nusselt number (Nur) decreases with an increase in any of the buoyancy-ratio number (Nr), the Brownian motion parameter (Nb), or the thermophoresis parameter (Nt). This correlation agrees well with eq. (32) in A.V. Kuznetsov & D. A. Nield [IV].

**V. Conclusions**

We present a numerical simulation on the steady, laminar, two-dimensional flow of a nanofluid along with an isothermal vertical plate. Nanofluid is considered as two components mixture. The solution procedures of the nonlinear partial differential equations of flow are discussed in detail. The associated computer codes in the Matlab environment are developed. After running these codes, dimensionless stream function (s), longitudinal velocity (s′), temperature (θ), and nanoparticle volume fraction (f) are computed and illustrated graphically for various values of five pertinent dimensionless parameters, namely, Prandtl number (Pr), Lewis number (Le), buoyancy-ratio parameter (Nr), Brownian motion parameter (Nb), and thermophoresis parameter (Nt). The present numerical outcome discloses that the reduced Nusselt number is a decreasing function of each of the numbers Nr, Nt, and Nb. All the present results are in very good harmony with relevant papers published in the literature.

**Conflict of Interest:**

There was no relevant conflict of interest regarding this paper.

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